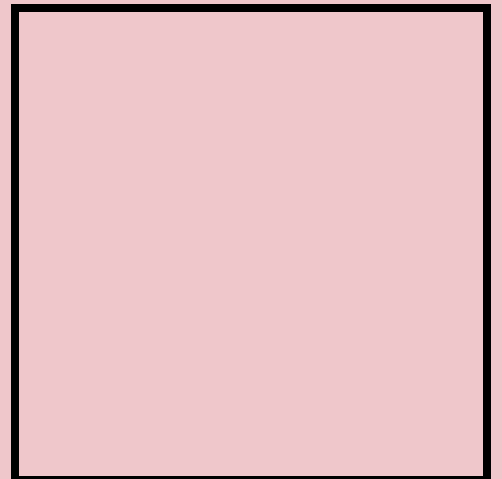


# Dimensionles Groups

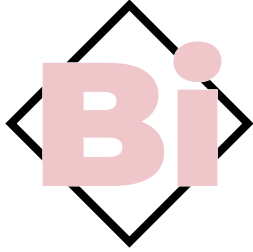
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**Definitions and Usage**

*February 2019*



## BIOT NUMBER



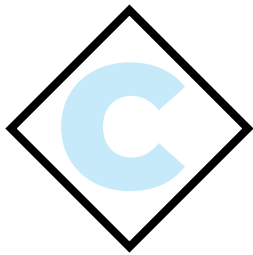
$\frac{\text{Heat Transfer to the Fluid}}{\text{Heat Transfer within the Fluid}}$

$$\text{Bi} = \frac{hl}{k}$$

*h* — convective heat transfer coefficient ( $\text{W}/\text{m}^2\text{K}$ )  
*l* — characteristic length (m)  
*k* — conductive heat transfer coefficient ( $\text{W}/\text{m K}$ )

**Modelling Application:** Transient heat transfer, and to identify dominant heat transfer mode. Biot  $< 0.1$  implies heat conduction within the body is much faster than the heat conduction away from the surface, and a lumped-capacitance model would have less than 5% error.

## CAUCHY NUMBER



$\frac{\text{Inertia Force}}{\text{Elastic Force}}$

$$\text{C} = \frac{\rho v^2}{p}$$

$\rho$  — fluid density ( $\text{kg}/\text{m}^3$ )  
 $v$  — fluid velocity (m/s)  
 $p$  — fluid pressure (Pa)

**Modelling Application:** Measure of compressibility, however the Mach number is more often used. For solids pressure should be replaced by the Young's Modulus of elasticity.

## CAVITATION NUMBER



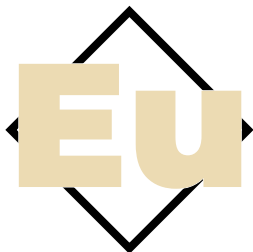
$\frac{\text{Pressure Force}}{\text{Inertia Force}}$

$$\text{Ca} = \frac{p - p_v}{\frac{1}{2} \rho v^2}$$

$p$  — fluid pressure (Pa)  
 $p_v$  — fluid vapor pressure (Pa)  
 $v$  — fluid velocity (m/s)  
 $\rho$  — fluid density ( $\text{kg}/\text{m}^3$ )

**Modelling Application:** Modeling cavitation phenomenon.

## EULER NUMBER



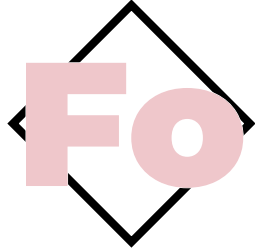
$\frac{\text{Pressure Force}}{\text{Inertia Force}}$

$$\text{Eu} = \frac{\Delta p}{\frac{1}{2} \rho v^2}$$

$\Delta p$  — local free stream pressure (Pa)  
 $v$  — free stream velocity (m/s)  
 $\rho$  — free stream density ( $\text{kg}/\text{m}^3$ )

**Modelling Application:** Fluid motion: this is half of the pressure coefficient, and is also similar to the cavitation number.

### FOURIER NUMBER



$$\frac{\text{Energy Conducted}}{\text{Energy Stored}}$$

$$Fo = \frac{\alpha t}{l^2}$$

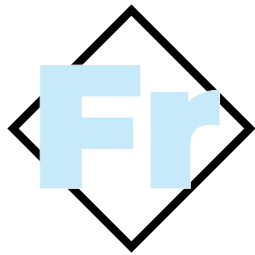
$\alpha$  - heat diffusivity ( $m^2/s$ )  
 $t$  - time (s)  
 $l$  - characteristic length (m)

$$Fo = \frac{kt}{\rho C_p l^2}$$

$k$  - conduction heat transfer ( $W/m K$ )  
 $C_p$  - specific heat capacity ( $J/kg K$ )  
 $\rho$  - density ( $kg/m^3$ )

**Modelling Application:** Analysis of transient heat transfer.

### FROUDE NUMBER



$$\frac{\text{Inertia Force}}{\text{Gravity Force}}$$

$$Fr = \frac{v^2}{gl}$$

$v$  - fluid velocity (m/s)  
 $g$  - gravitational acceleration ( $m/s^2$ )  
 $l$  - characteristic length (m)

**Modelling Application:** Analysis of free surface flows (ships), and open channel flow.

### GRAETZ NUMBER



$$\frac{\text{Thermal Capacity}}{\text{Conductive Heat Transfer}}$$

$$Gz = \frac{\dot{m} c_p}{kl}$$

$\dot{m}$  - mass flow rate (kg/s)  
 $c_p$  - specific heat capacity ( $J/kg K$ )  
 $k$  - conductive heat transfer coefficient ( $W/m K$ )  
 $l$  - characteristic length (m)

**Modelling Application:** Conductive heat transfer.

### GRASHOF NUMBER



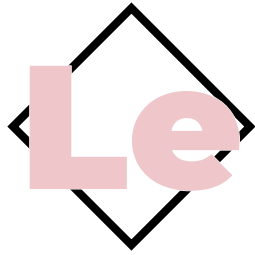
$$\frac{\text{Buoyancy Force}}{\text{Viscosity Force}}$$

$$Gr = \frac{g \beta \Delta T l^3}{\nu^2}$$

$g$  - gravitational acceleration ( $m/s^2$ )  
 $\beta$  - coefficient of cubic expansion ( $1/K$ )  
 $\Delta T$  - temperature difference (K)  
 $l$  - characteristic length (m)  
 $\nu$  - kinematic viscosity ( $m^2/s$ )

**Modelling Application:** Natural convection.

### LEWIS NUMBER



$$\frac{\text{Mass Diffusivity}}{\text{Thermal Diffusivity}}$$

$$\text{Le} = \frac{D}{\alpha}$$

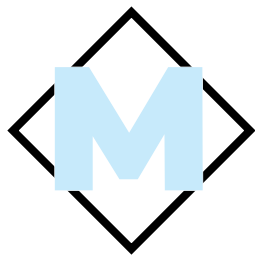
*D* — mass diffusivity (m<sup>2</sup>/s)  
*α* — heat diffusivity (m<sup>2</sup>/s)

$$\text{Le} = \frac{D}{k/\rho C_p}$$

*k* - conductive heat transfer (W/m K)  
*C<sub>p</sub>* - specific heat capacity (J/kg K)  
*ρ* - fluid density (kg/m<sup>3</sup>)

**Modelling Application:** Study of turbulent flow, flow transitions, and diffusion.

### MACH NUMBER



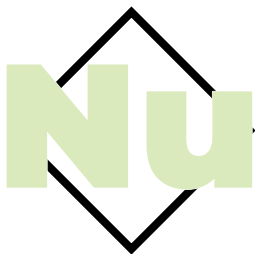
$$\sqrt{\frac{\text{Inertia Force}}{\text{Elastic Force}}}$$

$$\text{M} = \frac{v}{a}$$

*v* — fluid velocity (m/s)  
*a* — local speed of sound in the fluid (m/s)

**Modelling Application:** Analysis of compressible fluids, transonic flow, and supersonic flow.

### NUSSELT NUMBER



$$\frac{\text{Total Heat Transfer}}{\text{Conductive Heat Transfer}}$$

$$\text{Nu} = \frac{hl}{k}$$

*h* — convective heat transfer coefficient (W/m<sup>2</sup> K)  
*l* — characteristic length (m)  
*k* — conductive heat transfer coefficient (W/m K)

**Modelling Application:** Convective heat transfer.

### PECLET NUMBER



$$\frac{\text{Bulk Mass Transfer}}{\text{Diffusive Mass Transfer}}$$

$$\text{Pe} = \frac{v l}{D}$$

*v* — fluid velocity (m/s)  
*l* — characteristic length (m)  
*D* — mass diffusivity (m<sup>2</sup>/s)

**Modelling Application:** Diffusion and mass transfer, however if *D* is replaced with *α* this then becomes the heat convected divided by the heat conducted (i.e the Peclet heat transfer number).

## PRANDTL NUMBER



Momentum Diffusivity  
Thermal Diffusivity

$$\text{Pr} = \frac{\nu}{\alpha}$$

$\nu$  — kinematic viscosity ( $\text{m}^2/\text{s}$ )  
 $\alpha$  — heat diffusivity ( $\text{m}^2/\text{s}$ )

$$\text{Pr} = \frac{c_p \mu}{k}$$

$k$  - conduction heat transfer ( $\text{W/m K}$ )  
 $c_p$  - specific heat capacity ( $\text{J/kg K}$ )  
 $\mu$  - dynamic viscosity ( $\text{Pa s}$ )

**Modelling Application:** Boundary layer studies.

## RAYLEIGH NUMBER



Gravity Diffusivity  
Thermal Diffusivity

$$\text{Ra} = \frac{g \beta \Delta T l^3}{\nu \alpha}$$

$g$  — gravitational acceleration ( $\text{m/s}^2$ )  
 $\beta$  — coefficient of cubic expansion ( $1/\text{K}$ )  
 $\Delta T$  — temperature difference ( $\text{K}$ )  
 $l$  — characteristic length ( $\text{m}$ )  
 $\nu$  — kinematic viscosity ( $\text{m}^2/\text{s}$ )  
 $\alpha$  — heat diffusivity ( $\text{m}^2/\text{s}$ )

**Modelling Application:** Natural convection.

## REYNOLDS NUMBER



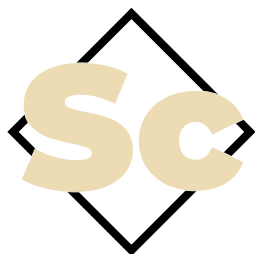
Inertia Force  
Viscosity Force

$$\text{Re} = \frac{\rho v l}{\mu}$$

$\rho$  — fluid density ( $\text{kg/m}^3$ )  
 $v$  — fluid velocity ( $\text{m/s}$ )  
 $l$  — characteristic length ( $\text{m}$ )  
 $\mu$  — dynamic viscosity ( $\text{Pa s}$ )

**Modelling Application:** Studying velocity profiles and characterizing fluid flows.

## SCHMIDT NUMBER



Momentum Diffusivity  
Mass Diffusivity

$$\text{Sc} = \frac{\nu}{D}$$

$\nu$  — kinematic viscosity ( $\text{m}^2/\text{s}$ )  
 $D$  — mass diffusivity ( $\text{m}^2/\text{s}$ )

**Modelling Application:** Boundary layer flow.

### STANTON NUMBER



$\frac{\text{Heat Transferred to Fluid}}{\text{Heat Transported by Fluid}}$

$$St = \frac{h}{\rho v C_p}$$

$h$  convective heat transfer coefficient ( $W/m^2 K$ )  
 $\rho$  fluid density ( $kg/m^3$ )  
 $v$  fluid velocity ( $m/s$ )  
 $C_p$  specific heat capacity ( $J/kg K$ )

**Modelling Application:** Convective heat transfer.

### STEFAN NUMBER



$\frac{\text{Heat Radiated}}{\text{Heat Conducted}}$

$$Sf = \frac{\eta A_r T^4}{k A_c \Delta T / \Delta l}$$

$\eta$  Stefan-Boltzmann ( $5.670 \times 10^{-8} J/K^4 m^2 s$ )  
 $A_r$  radiation area ( $m^2$ )  
 $T$  temperature ( $K$ )  
 $k$  conduction coefficient ( $W/m K$ )  
 $A_c$  conduction area ( $m^2$ )  
 $\Delta T / \Delta l$  conduction temperature gradient ( $K/m$ )

**Modelling Application:** Radiation heat transfer.

### STOKES NUMBER



$\frac{\text{Viscosity Force}}{\text{Gravity Force}}$

$$So = \frac{\mu v}{\rho g l^2}$$

$\mu$  dynamic viscosity ( $Pa s$ )  
 $v$  fluid velocity ( $m/s$ )  
 $\rho$  fluid density ( $kg/m^3$ )  
 $g$  gravitational acceleration ( $m/s^2$ )  
 $l$  characteristic length ( $m$ )

**Modelling Application:** Study particles moving through viscous flows.

### STROUHAL NUMBER



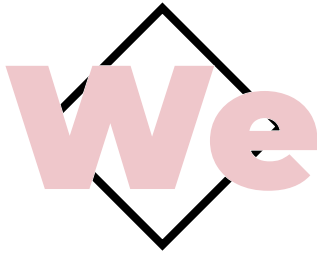
$\frac{\text{Vibration Speed}}{\text{Flow Speed}}$

$$Sr = \frac{\omega l}{v}$$

$\omega$  frequency of oscillation ( $rad/s$ )  
 $l$  characteristic length ( $m$ )  
 $v$  fluid velocity ( $m/s$ )

**Modelling Application:** Vortex phenomenon and aeroelasticity.

## WEBER NUMBER



$$\frac{\text{Inertia Force}}{\text{Surface Tension Force}}$$

$$We = \frac{\rho v^2 l}{\sigma}$$

$\rho$ : fluid density (kg/m<sup>3</sup>)  
 $v$ : fluid velocity (m/s)  
 $l$ : characteristic length (m)  
 $\sigma$ : surface tension (N/m)

**Modelling Application:** Free surface studies.

## THEORY

“Dimensionless numbers in many fields of engineering are collections of variables that provide order-of-magnitude estimates about the behavior of a system. They are often derived by combining coefficients from differential equations and are oftentimes a ratio between two physical quantities.”

- S. Ted Oyama, in *Membrane Science and Technology*, 2011

### Fundamental Dimensions

|             |          |                     |
|-------------|----------|---------------------|
| length      | <b>l</b> | m (meter)           |
| mass        | <b>m</b> | kg (kilogram)       |
| time        | <b>t</b> | s (second)          |
| temperature | <b>T</b> | °C (degree Celcius) |
| current     | <b>I</b> | A (Ampere)          |

A common dimensionless number in fluid dynamics is the Reynolds number. This ratio of inertial to viscous forces was correlated with experimental evidence to describe turbulent (Re > 2900) and laminar (Re < 2000) flow in pipes. Dimensional analysis generally enables predictability in the modeling of systems, for example enabling miniature ship models to represent the motion of larger vessels.

## DERIVATION

The formation of dimensionless groups is formalized as the **Buckingham Pi Theorem**, stating: If there are k variables and parameters in a problem and these contain r fundamental dimensions (e.g. m, l, t) the equation governing the system will have (k - r) dimensionless groups (Π groups). Further analysis determines if any group is useful. An excellent overview can be found [here](#), summary below.

**1** Choose parameters likely to affect variables (e.g. the force of drag (F)).

$$\frac{m}{l^2} \rightarrow F, \mu, \rho, v, D$$

$$\frac{m}{l^2} \rightarrow k = 5$$

**2** Select “repeating” terms (e.g. ρ, D, v), used to balance out “non-repeating” terms (e.g. F, μ).

$$\frac{m}{l^3} \rightarrow \rho, D, v \leftarrow \frac{l}{t}$$

**3** Count the number of fundamental dimensions that need to be considered.

$$m, l, t$$

$$r = 3$$

**4** Calculate the number of possible Π groups, these are dimensionless groups.

$$k - r = 2$$

**5** Form Π group.

$$\Pi_1 = \mu \rho^a v^b D^c$$

$$\Pi_1 = \frac{m}{l^2} \left(\frac{m}{l^3}\right)^a \left(\frac{l}{t}\right)^b l^c$$

$$m: 0 = -1 + a \quad a = 1$$

$$l: 0 = -1 - 3a + b + c \quad c = -1$$

$$t: 0 = -1 - b \quad b = -1$$

$$\Pi_1 = \mu / (\rho v l)$$

## USAGE

If a system consists of small particles falling through a viscous medium, and it is characterized by requiring that the ratio between gravity and inertia forces remain constant, how would the drag associated with the system scale with respect to system size?

Hold dimensionless group ratio constant & solve for velocity ratio

$$\frac{\text{Inertia Force}}{\text{Gravity Force}} = Fr = \frac{v^2}{gl}$$

$$\frac{v_1^2}{g l_1} = \frac{v_0^2}{g l_0}$$

$$\frac{v_1^2}{v_0^2} = \frac{g l_1}{g l_0} \rightarrow \frac{v_1}{v_0} = \sqrt{\frac{l_1}{l_0}}$$

Create drag ratio & solve for length by plugging in velocity

$$\frac{D_1}{D_0} = \frac{6\pi\mu l_1 v_1}{6\pi\mu l_0 v_0}$$

$$\frac{D_1}{D_0} = \frac{l_1}{l_0} \frac{\sqrt{l_1}}{\sqrt{l_0}} = \frac{l_1^{3/4}}{l_0^{3/4}}$$

This ratio can now be used to scale a model for testing such that the ratio of inertia and gravity forces (Freud Number) is constant. Any length scaling must be set to the 3/4 power to represent the scaling of associated drag.

|                            |  |
|----------------------------|--|
| <b>Ar</b>                  | <b>Archimedes Number</b> Buoyancy, entrainment, and two-phase flow             |
| <b><math>\alpha</math></b> | <b>Arrhenius Number</b> Chemical reactions, and energy analysis                |
| <b>Bm</b>                  | <b>Bingham Number</b> Viscoelastic, plastic, and non-Newtonian flow            |
| <b>Bl</b>                  | <b>Blake Number</b> Capillary, and two-phase flow                              |
| <b>Bs</b>                  | <b>Bodenstein Number</b> Mass and momentum transfer                            |
| <b>Bo</b>                  | <b>Bond Number</b> Capillary flow, and surface tension (Eotvos Number, $E_o$ ) |
| <b>Br</b>                  | <b>Brinkman Number</b> Organic liquid flow                                     |
| <b>J</b>                   | <b>Colburn Number</b> Momentum and mass diffusivity                            |
| <b>De</b>                  | <b>Dean Number</b> Non-straight line flow                                      |
| <b>E</b>                   | <b>Eckert Number</b> Energy dissipation  |
| <b>Ga</b>                  | <b>Galileo Number</b> Flow against gravity                                     |
| <b>H</b>                   | <b>Hodgson Number</b> Pulsating flow   |
| <b>Ja</b>                  | <b>Jakob Number</b> Heat transfer  |
| <b>Kn</b>                  | <b>Knudsen Number</b> Compressible flows                                       |
| <b>Z</b>                   | <b>Ohnesorge Number</b> Capillary flow, and surface tension                    |



|       |   |
|-------|---|
| $Sh$  | <b>Sherwood Number</b> Mass transfer and heat transfer  |
| $So$  | <b>Sommerfeld Number</b> Lubrication and bearing design |
| $C_d$ | <b>Drag Coefficient</b> Fluid flow over objects         |
| $f$   | <b>Friction Factor</b> Fluid flow over surfaces         |
| $N_p$ | <b>Power Number</b> Turbomachinery                      |
| $C_p$ | <b>Pressure Coefficient</b> Fluid flow over surfaces    |

Equations for these additional numbers can be found here: <https://www.iist.ac.in/sites/default/files/people/numbers.html>

Thank you to **Dr. Graham Walker** for the compilation of these Dimensionless Groups and the example question.